International Journal of Applied and Natural Sciences (IJANS)
ISSN(P): 2319-4014; ISSN(E): 2319-4022
Vol. 6, Issue 4, Jun – Jul 2017; 113-128

International Academy of Science, Engineering and Technology
Connecting Researchers; Nurturing Innovations

A MATHEMATICAL MODEL TO STUDY THE IMPACT OF AWARENESS PROGRAM FOR SUSTAINABLE MANAGEMENT OF RESOURCE BIOMASS BY TRIBAL POPULATION

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ABSTRACT

There is the largest concentration of forest resources in India along with equally large concentration of tribal and rural population. Tribal population and rural population are highly dependent on forest resources for their livelihood. Degradation and depletion of forest resources, increase poverty and suffering among the rural population, therefore it is imperative to educate the tribal population to use the forest products efficiently and make efforts for sustainable management of forest resource biomass. Proper awareness of tribal population is demand of the time. Keeping these objectives in mind, we have formulated a mathematical model consisting of biomass, tribal population, trained tribal population, awareness programme to aware and train the tribal population for efficient use of forest products. Local and global stability analysis of the mathematical model along with the persistence of the system is checked using the theory of nonlinear ordinary differential equations. Analytical results obtained are justified numerically through numerical simulation. The important parameters are investigated and a variety of variables to change in these parameters is determined.

KEYWORDS: Forestry Biomass, Population, Stability, Persistence

1. INTRODUCTION

We are living in the twenty first century. The world has seen spectacular political, social, cultural, economic and scientific progress in this century. But when we look at cost paid by us to achieve this progress it seems that what we have gained is much less than the loss faced by the environment due to its over exploitation. Over the years, there has been progressive pressure on the environment and the natural resources, the alarming consequences of which are becoming evident in the form of frequent natural disasters in the form of earthquakes, droughts and floods throughout the world. These consequences have direct impact on the lives of the poor who are directly dependent on natural resources. Demographic and economic growth has historically driven deforestation, forest exploitation, and agricultural demand (South, 1999; Wright and Muller-Landau, 2006; Cohen, 2014a, b), and such growth will continue in the developing world. Our forest wealth is deteriorating due to over-grazing, over-exploitation both for commercial and household needs, encroachments by the tribal and rural population. Economic growth in the developing world is projected to double global consumption of forest products by 2030 (WWF and IIASA, 2012: Ch4. p. 9). Other unsustainable practices include shifting cultivation and other developmental activities such as roads, buildings, irrigation and power projects in the rural areas as well.

On one side there are deforestation and an increase in unsustainable practices in rural areas, the other side includes an ever increasing movement of poor families to towns. In fact, Lack of opportunities for employment in villages has led to an ever increasing movement of resource-poor families in the town leading in expansion of urban slum areas. Illiteracy and child labour are persisting in urban areas. Records say that the global rate of forest loss has decreased since 2010 to 3.3 million hectares (Mha) or 0.08% annually (Keenan et al., 2015). Similarly, rates of afforestation are steady or rising not just in temperate countries, were planted forests have long been integral elements of the forest estate, but also in the tropics, where the extent of planted forests has nearly doubled since 1990 (Payn et al., 2015). Notwithstanding this progress, forest conversion for agriculture has not reduced and may lead to eventual forest loss (Van Lierop and Lindquist, 2015).

Hence, seeing this scenario, awareness or educating the masses, especially in the tribal and rural population is one of the main tools that will help to perform transition to a greener Earth and saving the environment from continuous deterioration. Awareness helps in disseminating innovations in technology to the people who need it in actual. It is the ability of the society to generate new ideas and approaches to solve environmental related problems. The demand for new skills is growing and in order to accomplish this task, the systems of education and professional training will need to provide well-trained and highly qualified personnel.

Keeping all these things in mind, we formulate a nonlinear mathematical model using ordinary differential equations, considering different equations for resource biomass, tribal population, aware or educated tribal population and awareness programme. There have been many studies investigating the impact of industrialization and increasing population pressure on the environment. Many researchers have investigated the depletion of resource biomass by overgrowing population, toxicants and industrialization. Shukla et al (1989, 1996, and 1997) has studied the effects of population and pollution on the depletion of forestry resources. Dubey and Dass (1999) have proposed and analyzed a mathematical model to study the survival of species dependent on a resource, which is depleted due to industrialization. Dubey and Narayanan (2010) studied a mathematical model to demonstrate the effect of industrialization, population, and pollution on the depletion of a renewable resource. M. Agarwal et al. (2010) proposed a ratio dependent mathematical model on the depletion of forestry resource biomass due to industrialization pressure. Shukla et al. (2003) studied the effects of primary and secondary toxicants on the resource biomass. Shukla et al. (2011) studied the effect of technology on the conservation of forestry resource biomass. It was further observed that in the tropics, where most forest change has occurred, deforestation due to smallholder agricultural expansion has given way to large-scale, enterprise-driven forest conversion (Hecht, 2005; Rudel, 2007; Asner et al., 2013). Sayer and Cassman (2013) argued that future agricultural demand for land could be met without significant forest loss if agricultural innovations were deployed. Sloan and Sayer (2015) gave a review of Forest Resources Assessment of 2015 showing positive global trends, but still it was observed that forest loss and degradation persist in poor tropical countries. Going through the above literature, it was observed that none of them considered tribal and trained tribal population along with the effect of awareness on them for sustainable management of our resources, which is novelty our work. We aim at finding the impact of training tribal population of conservation of resource biomass. Our paper is classified as follows: In section one brief introduction about the problem is given, in section two mathematical models are given. Section three includes boundedness of the system, section four; five and six illustrate equilibrium, stability (both local and global) analysis and persistence of the model. In section seven and eight numerical simulations and conclusion of the model respectively is given, followed by the list of references.

2. MATHEMATICAL MODEL

Let B is biomass density of forest resources, P is tribal population, T is aware tribal population and A is an awareness programme to train the tribal population. It is assumed that resource biomass are growing in intrinsic growth rate r, tribal population is consuming the resourc.iomass at the rate α. r_0 is the rate at which resource biomass is benefited by the awareness program launched among the tribal population. The intrinsic growth rate of tribal population is s. Carrying capacity of resource biomass and tribal population is given by K and K_1 respectively. The rate at which tribal population gets awareness about sustainable consumption of forest resources is γ. s_0 and γ_0 are the migration rate of tribal and aware tribal population to the urban areas in search of better employment opportunities. η is the rate at which awareness programme is launched, Awareness programme is launched proportional to the tribal or rural population of the system. η_0 is the failure rate of awareness program.

$$\begin{aligned}
\frac{dB}{dt} &= rB\left(1-\frac{B}{K}\right)-\alpha BP - m\alpha BT,\\
\frac{dP}{dt} &= sP\left(1-\frac{P}{K_1}\right)+\alpha BP - \gamma P A - \mu_1 P,\\
\frac{dT}{dt} &= m\alpha TB + \gamma P A - \mu_2 T,\\
\frac{dA}{dt} &= \eta P - \eta_1 A - \eta_0 T.
\end{aligned} \tag{2.1}$$

Where $B(0) > 0, P(0) \ge 0, T(0) \ge 0, A(0) \ge 0.$

3. BOUNDEDNESS OF THE SYSTEM

In analogy to the population dynamics, it is very important to observe the consequences that restrict the growth of the population. In this sense, the study of boundedness of the solution of system around different steady states is very much needed. For this, we find boundedness on the system in the following lemma:

Lemma *3.1:* The set $\Omega = \{(B,P,T,A): 0 \le B \le K, 0 \le P \le P_m, 0 \le \mathrm{T} \le \mathrm{T_m} \text{ and } 0 \le A \le A_m\}$,

Where

$$P_m = \frac{(s-\mu_1+\alpha K)K_1}{s}, T_m = \frac{\gamma P_m A_m}{\mu_2 - m\alpha K} \text{ and } A_m = \frac{\eta P_m}{\eta_1},$$

with conditions $s+\alpha K > \mu_1, \mu_2 > m\alpha K$

is the region of attraction for all solutions initiating in the interior of the positive octant.

Proof: Let (B,P,T,A) be solution with positive initial values (B_0, P_0, T_0, A_0).Define a function

From the first equation of the system (2.1)

$$\frac{dB}{dt} \le rB\left(1 - \frac{B}{K}\right),$$

$$\frac{dB}{dt} \le K,$$

According to comparison principle, it follows that

$$B_m = K.$$

From the second equation of the system (2.1), we get

$$\frac{dP}{dt} \le sP\left(1 - \frac{P}{K_1}\right) + \alpha\, K\, P - \gamma\, P\, A - \mu_1 P,$$

$$\frac{dP}{dt} \le (s + \alpha\, K - \mu_1)P - \frac{sP^2}{K_1},$$

According to comparison principle, we have

$$P_m = \frac{(s + \alpha\, K - \mu_1)K_1}{s},$$

with condition

$$s + \alpha\, K > \mu_1.$$

From the forth equation of the system (2.1), we find it

$$\frac{dA}{dt} \le \eta P_m - \eta_1 A.$$

According to comparison principle, we get

$$A_m = \frac{\eta P_m}{\eta_1}.$$

From the third equation of the system (2.1), we get

$$\frac{dT}{dt} \le m\alpha TK + \gamma\, P_m\, A_m - \mu_2 T,$$

$$\frac{dT}{dt} \le \gamma\, P_m\, A_m - (\mu_2 - m\alpha K)T,$$

$$T_m = \frac{\gamma\, P_m A_m}{\mu_2 - m\alpha K},$$

with condition

$\mu_2 > m\alpha K.$

Therefore, all solutions of the system (2.1) enter into the region

$$\Omega = \{(B, P, T, A): 0 \le B \le K, 0 \le P \le P_m, 0 \le \mathrm{T} \le \mathrm{T_m} \text{ and } 0 \le A \le A_m\},$$

where

$$P_m = \frac{(s - \mu_1 + \alpha K)K_1}{s}, T_m = \frac{\gamma P_m A_m}{\mu_2 - m\alpha K} \text{ and } A_m = \frac{\eta P_m}{\eta_1},$$

With conditions $s + \alpha K > \mu_1, \mu_2 > m\alpha K$

is the region of attraction for all solutions initiating in the interior of the positive octant.

This completes the proof of the lemma.

In the next Section we present the equilibrium analysis of the model.

4. EQUILIBRIUM ANALYSIS

The system (2.1) has four nonnegative equilibria namely; $E_2\left(0, \overline{P}, \overline{T}, \overline{A}\right)$ and $E_3\left(B^*, P^*, T^*, A^*\right)$.The existing of equilibrium points $E_0(0,0,0,0)$ and $E_1(K,0,0,0)$, is obvious. We show the existence of other equilibria as follows

The existence of $E_2\left(0, \overline{P}, \overline{T}, \overline{A}\right)$

Here $\overline{P}, \overline{T}$ and $\overline{A}$ are the positive solutions of the system of algebraic equations given below

$$s\left(1 - \frac{\overline{P}}{K_1}\right) - \gamma \overline{A} - \mu_1 = 0, \tag{4.1}$$

$$\gamma \overline{P}\, \overline{A} - \mu_1 \overline{T} = 0, \tag{4.2}$$

$$\eta \overline{P} - \eta_1 \overline{A} - \eta_0 \overline{T} = 0. \tag{4.3}$$

From the equations (4.1) and (4.2), we get

$$\overline{P} = \frac{\left(s - \mu_1 - \gamma \overline{A}\right)K_1}{s}. \tag{4.4}$$

with condition

$$s > \mu_1 + \gamma \overline{A}. \tag{4.5}$$

$$\overline{T} = \frac{\gamma K_1\left(s - \mu_1 - \gamma \overline{A}\right)\overline{\mathrm{A}}}{\mu_1}. \tag{4.6}$$

Using the values of $\overline{P}$ and $\overline{T}$ from the equations (4.4), (4.6) with condition (4.5) in equation (4.1), we get

$$\overline{A} = \frac{A_2 \pm \sqrt{A_2^2 - 4A_1A_3}}{2A_1}. \tag{4.7}$$

Where

$$A_1 = s\eta_0\gamma^2 K_1, A_2 = \gamma\eta K_1\mu_1 + \mu_1 s\eta_1 + \eta_0\gamma K_1 s(s-\mu_1), A_3 = \mu_1\eta K_1(s-\mu_1). \tag{4.8}$$

This completes the existence of E_2.

The existence of $E_3(B^*, P^*, T^*, A^*)$

Here B^*, P^*, T^* and A^* are the positive solutions of the system of algebraic equations given below

$$r\left(1-\frac{B^*}{K}\right) - \alpha P^* - m\alpha T^* = 0, \tag{4.9}$$

$$s\left(1-\frac{P^*}{K_1}\right) + \alpha B^* - \gamma A^* - \mu_1 = 0, \tag{4.10}$$

$$m\alpha T^* B^* + \gamma P^* A^* - \mu_2 T^* = 0, \tag{4.11}$$

$$\eta P^* - \eta_1 A^* - \eta_0 T^* = 0. \tag{4.12}$$

From equation (4.9), we get

$$P^* = \frac{r}{\alpha}\left(1-\frac{B^*}{K}\right) - m\alpha T^* = f(B^*, T^*). \tag{4.13}$$

From equation (4.10), we have

$$A^* = \frac{1}{K_1\alpha\gamma}\left\{s\alpha K_1 - sr + B^*\left(\frac{sr}{K} + K_1\alpha^2\right) - \mu_1 K_1\alpha + sm\alpha^2 T^*\right\} = f_1(B^*, T^*). \tag{4.14}$$

Using the equations (4.13) and (4.14) in the equations (4.11) and (4.12), we have following equations

$$\begin{aligned} H_1(B^*, T^*) = {} & B^*T^*\left\{K_1 m\alpha^2 - \frac{rsm\alpha}{K} - m\alpha\left(\frac{rs}{K} + K_1\alpha^2\right)\right\} + T^*\{-\mu_1 K_1\alpha + rsm\alpha - m\alpha(sK_1\alpha - sr - \mu_1 K_1\alpha)\} \\ & + B^*\left\{\frac{r}{\alpha}\left(\frac{sr}{K} + K_1\alpha^2\right) + \frac{r}{\alpha K}(sK_1\alpha - sr - \mu_1 K_1\alpha)\right\} - \frac{r}{\alpha K}\left(\frac{sr}{K} + K_1\alpha^2\right)B^{*2} - m^2\alpha^3 sT^{*3} + \\ & \frac{r}{\alpha}(sK_1\alpha - sr - \mu_1 K_1\alpha). \end{aligned} \tag{4.15}$$

$$\begin{aligned} H_2(B^*, T^*) = {} & \frac{\eta r}{\alpha}\left(1-\frac{B^*}{K}\right) - \eta m\alpha T^* - \frac{\eta_1}{K_1\alpha\gamma}\{sK_1\alpha - sr + B^*\left(\frac{sr}{K} + K_1\alpha^2\right) - \mu_1 K_1\alpha \\ & + sm\alpha^2 T^*\} - \eta_0 T^*. \end{aligned} \tag{4.16}$$

From equation (4.15) we note the following

When $T^* \to 0$, then $B^* \to B_a^*$ where

$$C_1 B_a^{*2} + C_2 B_a^* + C_3 = 0. \quad \textbf{(4.17)}$$

Where

$$C_1 = \frac{r}{\alpha K}\left(\frac{sr}{K} + K_1\alpha^2\right), C_2 = KC_1 - \frac{r}{\alpha K}\left(sK_1\alpha - sr - \mu_1 K_1\alpha\right), C_3 = K^2 C_1 - KC_2.$$

Thus equation (4.17) has unique positive solution if following conditions hold

$$\frac{r}{\alpha K}\left(sK_1\alpha - sr - \mu_1 K_1\alpha\right) < KC_1 < C_2, sK_1\alpha > sr + \mu_1 K_1\alpha. \quad \textbf{(4.18)}$$

$$\frac{\partial H_1}{\partial T^*} = B^* B_1 + T^* B_2 + B_3. \quad \textbf{(4.19)}$$

$$\frac{\partial H_1}{\partial B^*} = B^* B_4 + T^* B_5 + B_6. \quad \textbf{(4.20)}$$

Where

$$B_1 = K_1 m\alpha^2 - \frac{rsm\alpha}{K} - m\alpha\left(\frac{sr}{K} + K_1\alpha^2\right), B_2 = -2m^2\alpha^3 s,$$

$$B_3 = -\mu_1 K_1\alpha + rms\alpha + \left(\frac{r}{\alpha} - m\alpha\right)\left(sK_1\alpha - sr - \mu_1 K_1\alpha\right), B_4 = -\frac{2r}{\alpha K}\left(\frac{sr}{K} + K_1\alpha^2\right),$$

$$B_5 = K_1 m\alpha^2 - \frac{rsm\alpha}{K} - m\alpha\left(\frac{sr}{K} + K_1\alpha^2\right), B_6 = \frac{r}{\alpha}\left\{\frac{sr}{K} + K_1\alpha^2 + \frac{\left(sK_1\alpha - sr - \mu_1 K_1\alpha\right)}{K}\right\}.$$

Now from equations (4.19) and (4.20), we get

$$\frac{dB^*}{dT^*} = -\frac{\frac{\partial H_1}{\partial T^*}}{\frac{\partial H_1}{\partial B^*}}. \quad \textbf{(4.21)}$$

It is clear that $\frac{dB^*}{dT^*} > 0$, if either

i. $\frac{\partial H_1}{\partial B^*} > 0$ and $\frac{\partial H_1}{\partial T^*} < 0$, or **(4.22)**

ii. $\frac{\partial H_1}{\partial B^*} < 0$ and $\frac{\partial H_1}{\partial T^*} > 0$

hold

From equation (3.16) we get following

When $T^* \to 0$, then $B^* \to B_b^*$ where

$$B_b^* = \frac{\eta\, rKK_1\gamma + \eta_1 Ksr - \eta_1 KK_1\alpha(1+\mu_1)}{\eta_1\left(sr + KK_1\alpha^2\right) + \eta\, rK_1\gamma}. \quad \textbf{(4.23)}$$

With condition

$$\eta\, rKK_1\gamma + \eta_1 Ksr > \eta_1 KK_1\alpha(1+\mu_1). \quad \textbf{(4.24)}$$

$$\frac{\partial H_2}{\partial T^*} = -\eta m\alpha - \frac{\eta_1 sm\alpha^2}{K_1\alpha\gamma} - \eta_0. \quad \textbf{(4.25)}$$

$$\frac{\partial H_2}{\partial B^*} = -\frac{\eta\, r}{\alpha K} - \frac{\eta_1}{K_1\alpha\gamma}\left(\frac{sr}{K} + K_1\alpha^2\right). \quad \textbf{(4.26)}$$

Now from equations (4.25) and (4.26), we get

$$\frac{dB^*}{dT^*} = -\frac{\dfrac{\partial H_2}{\partial T^*}}{\dfrac{\partial H_2}{\partial B^*}}. \quad \textbf{(4.27)}$$

It is clear that $\dfrac{dB^*}{dT^*} > 0$, if either

i. $\dfrac{\partial H_2}{\partial B^*} > 0$ and $\dfrac{\partial H_2}{\partial T^*} < 0$, or **(4.28)**

ii. $\dfrac{\partial H_2}{\partial B^*} < 0$ and $\dfrac{\partial H_2}{\partial T^*} > 0$

hold.

From the above analysis, we note that two isoclines (4.15) and (4.16) interest in a unique $\left(B^*, T^*\right)$ if in addition to the conditions (4.22) and (4.28), the inequality $B_a^* < B_b^*$ holds. Knowing the value of B^* and T^*, the value of P^* and A^* can be calculated from equations (4.13) and (4.14). This completes the existence of $E_3\left(B^*, P^*, T^*, A^*\right)$.

5. LOCAL STABILITY

To discuss the local stability of the system (2.1), we compute the variational matrix of the system (2.1). The entries of the general vocational matrix are given by differentiating the right side of the system (2.1) with respect to B, P, T and, i.e.

$$V(E)=\begin{bmatrix} t_{11} & t_{12} & t_{13} & 0 \\ t_{21} & t_{22} & 0 & t_{24} \\ t_{31} & t_{32} & t_{33} & t_{34} \\ 0 & t_{42} & t_{43} & t_{44} \end{bmatrix}.$$

Where

$$t_{11}=r-\frac{2rB}{K}-\alpha P-m\alpha T,\ t_{12}=-\alpha B,\ \ t_{13}=-m\alpha B,\ t_{21}=\alpha P,\ t_{22}=s-\frac{2sP}{K_1}+\alpha B-\gamma A-\mu_1,$$
$$t_{24}=-\gamma P,\ t_{31}=m\alpha T,\ t_{32}=\gamma A,\ t_{33}=m\alpha B-\mu_2, t_{34}=\gamma P, t_{42}=\eta, t_{43}=-\eta_0, t_{44}=-\eta_1.$$

Accordingly, the linear stability analysis about the equilibrium points $E_i, i=0,1,2,3$ gives the following results:

1. The equilibrium point E_0 is unstable manifold in $B-P$ plane and stable manifold in $T-A$ plane.
2. The equilibrium point E_1 is unstable manifold in $P-$ direction and stable manifold in $B-T-A$ plane.
3. The equilibrium point E_2 is unstable manifold in $B-$ direction.

The stability behaviour of equilibrium point E_3 is not obvious. However, in the following theorem we give sufficient conditions for equilibrium points E_3 to be locally asymptotically stable. For proof, see Appendix A.

Theorem (5.1): Let the following inequalities hold:

$$(\gamma A^*)^2<\frac{2sT^*}{K_1}(\mu_2-m\alpha B^*), \tag{5.1}$$

$$(P^*-\eta_0 T^*)^2<\frac{2s\eta_1}{K_1\eta_0\gamma T^*}. \tag{5.2}$$

Then an equilibrium point E_3 is locally asymptotically stable.

Global Stability

The following theorem characterizes the global stability behavior of equilibrium point E_3.For proof, see Appendix B.

Theorem (5.2) Let the following inequalities hold:

$$m^2\alpha^2(\eta_0 T_m-P^*\eta)^2<\frac{2r\eta_0\eta P^*}{K}(\mu_2-m\alpha B^*), \tag{5.3}$$

$$\gamma^2 A_m^2<\frac{2s\eta P^*}{K_1\eta_0}(\mu_2-m\alpha B^*). \tag{5.4}$$

Then an equilibrium point E_3 is globally stable in the region Ω.

6. PERSISTENCE

Theorem (6.1): Assume that $r > \alpha P_m + m\alpha T_m, s + \alpha B_{\min} > \gamma A_m + \mu_1, \mu_2 > m\,\alpha\, B_{\min}$ and $\eta P_{\min} > \eta_0 T_m$. Here K, P_m, T_m and A_m are upper bounds of the populations B, P, T, A respectively and always positive. Then the system (1) becomes persists.

Proof: From the first equation of the system (1), we have

$$\frac{dB}{dt} \geq \left(r - \alpha\, P_m - m\alpha\, T_m\right)B - \frac{rB^2}{K}. \tag{6.1}$$

According to lemma (3.1) and comparison principle, it follows that

$$B_{\min} = \frac{K\left(r - \alpha\, P_m - m\alpha T_m\right)}{r}. \tag{6.2}$$

With condition $r > \alpha\, P_m + m\alpha T_m$, $B_{\min}$ remains always positive.

From the second equation of the system (1) and equation (6.2), we have

$$\frac{dP}{dt} \geq \left(s + \alpha B_{\min} - \gamma A_m - \mu_1\right)P - \frac{s}{K_1}P^2. \tag{6.3}$$

According to lemma (3.1) and comparison principle, it follows that

$$P_{\min} = \frac{\left(s + \alpha B_{\min} - \gamma A_m - \mu_1\right)K_1}{s}. \tag{6.4}$$

With condition $s + \alpha B_{\min} > \gamma A_m + \mu_1$, $P_{\min}$ remains always positive.

From the last equation of the system (1) equation (6.4), we have

$$\frac{dA}{dt} \geq \eta P_{\min} - \eta_0 T_m - \eta_1 A. \tag{6.5}$$

Using the comparison principle, it follows that

$$A_{\min} = \frac{\left(\eta\, P_{\min} - \eta_0 T_m\right)}{\eta_1}. \tag{6.6}$$

With condition $\eta\, P_{\min} > \eta_0 T_m$ $A_{\min}$ remains always positive.

From the third equation of the system (1) and equations (6.2), (6.4), (6.6), we get

$$\frac{dT}{dt} \geq \gamma\, P_{\min} A_{\min} - (\mu_2 - m\,\alpha\, B_{\min}). \tag{6.7}$$

According to lemma (3.1) and comparison principle, it follows that

$$T_{\min} = \frac{\gamma P_{\min} A_{\min}}{\mu_2 - m\alpha B_{\min}}. \tag{6.8}$$

With condition $\mu_2 > m\alpha B_{\min}$, $T_{\min}$ remains always positive.

This completes the proof of the theorem. Then the system (1.1) becomes persists.

7. NUMERICAL SIMULATION

The global stability of the non-linear model system (2.1), in the positive octant, is investigated numerically. The numerical integration of model systems (2.1) is carried out for various choices of biologically feasible parameter values and for different sets of initial conditions. In all the cases being considered here the data are chosen such that the persistence conditions are satisfied. Model system (2.1) is solved numerically using the Runge–Kutta method. The dynamic behavior and its corresponding time series of the model system (2.1) are decided on the following data set:

$$r = 10, K = 150, \alpha = 0.02, m = 0.001, s = 3, K_1 = 100, \gamma = 0.001, \mu_1 = 0.001, \mu_2 = 0.3, \eta = 0.03,$$
$$\eta_1 = 0.04, \eta_0 = 0.02. \tag{7.1}$$

The interior equilibrium point of the model system (2.1) corresponding to the feasible parameter values as (7.1) is:

$$E_3(100.80,\ 163.95,\ \ 53.05,\ 96.43).$$

Figures 1, 2, 3 and 4 are drawn to study important parameters in the system. Figure 1 is plotted to display the variation of resource biomass, tribal population, aware tribal population and awareness programme with time. It manifests the local stability of the system at an interior equilibrium point as each population approach the equilibrium value as time tends to infinity.

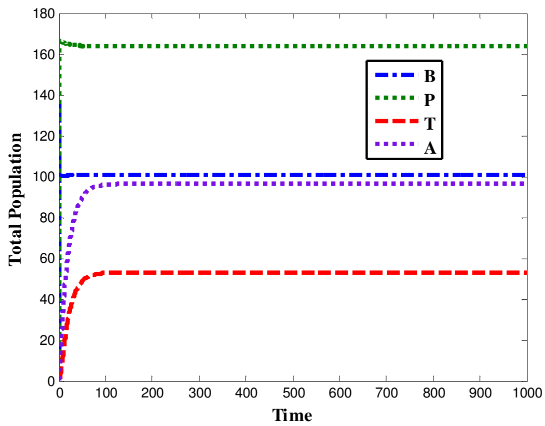

Figure 1

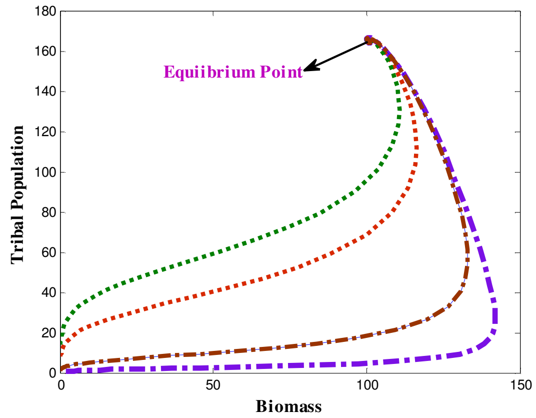

Figure 2

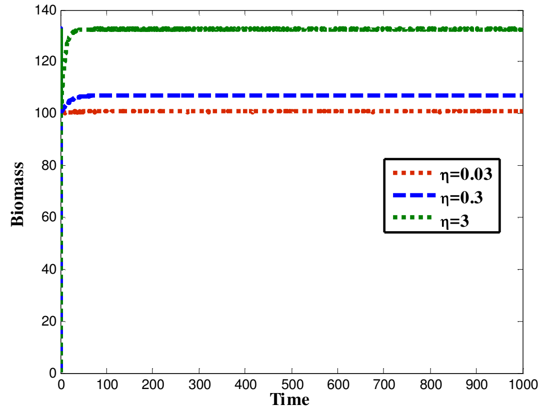

Figure 3

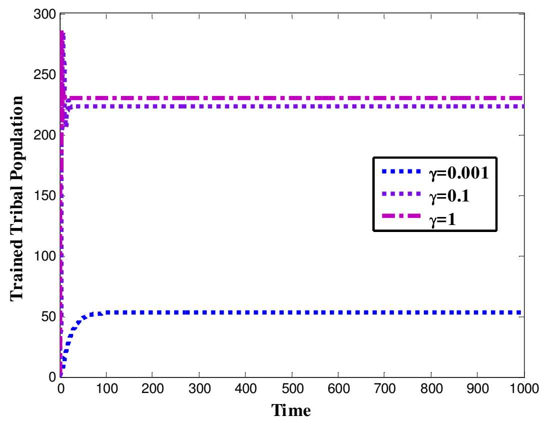

Figure 4

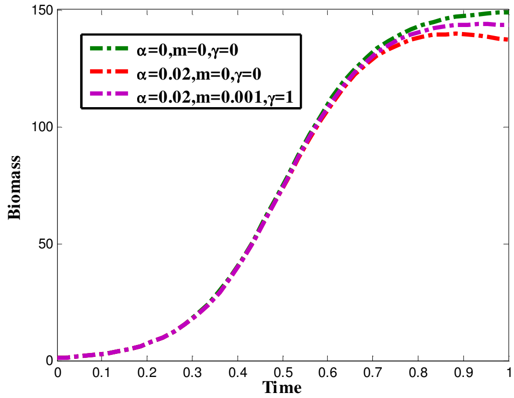

Figure 5

DISCUSSIONS AND CONCLUSIONS

Tribal population and rural population are highly dependent on forest resources for their livelihood. Degradation and depletion of forest resources, increase poverty and suffering among rural population, therefore it is imperative to educate the tribal population to use the forest products efficiently and make efforts for sustainable management of forest resource biomass. Proper awareness of tribal population is demand of the time. Keeping these things in mind, we propose and solve a mathematical model to study the impact of awareness program among tribal and rural population. Our study includes model formulation followed by qualitative analysis of the formulated nonlinear mathematical model. Qualitative analysis of the model includes Equilibrium analysis, local and global stability analysis and persistence of the system. Numerical simulation is performed to justify the analytical findings and graphs are plotted to study the variation of important variables of the system with time for different parameters. Analytically conditions for the local and global stability of the system are determined and these conditions are further justified numeric by considering a hypothetical set of parameter values. Local and global stability of the system is displayed through the graph also. Through the graph it is observed that as the rate of launch of awareness program in the system increases the equilibrium value of resource biomass increases. This implies that as people get awareness about sustainable management of resources, exploitation of resources decreases and hence the original equilibrium level of resource biomass increases. Thus, awareness program helps to increase the equilibrium level of resource biomass and it is imperative to be motivated both by the government and public. In addition, it is observed that the density of resource biomass decreases gradually by continuous use of resource biomass without any awareness program. However, when awareness program is launched then resource biomass retain its equilibrium value and approaches the same equilibrium level as if it is untouched by the tribal population.

ACKNOWLEDGEMENTS

Author Dr. Rachana Pathak thankfully acknowledges the NBHM (2/40 (29) /2014/R&D-11/14138) for the financial assistance in the form of PDF.

Appendix A. Proof of Theorem (5.1)

To prove Theorem, we first linearize model (1) by substituting

$$B = B^* + b, P = P^* + p, T = T^* + \tau, A = A^* + a.$$

Where b, p, τ, a are small perturbations around an equilibrium point E_3. We get a following linearize system

$$\frac{db}{dt} = -\frac{rB^*}{K}b - \alpha B^* p - m\alpha B^* \tau,$$

$$\frac{dp}{dt} = -\frac{sP^*}{K_1}p + \alpha P^* b - \gamma P^* a, \qquad \textbf{(A.1)}$$

$$\frac{d\tau}{dt} = m\alpha T^* b + \gamma A^* p + \gamma P^* a + \left(m\alpha B^* - \mu_2\right)\tau,$$

$$\frac{da}{dt} = \eta p - \eta_1 a - \eta_0 \tau.$$

Then we consider the following positive definite function:

$$W = \frac{1}{2}b^2 + \frac{n_1}{2}p^2 + \frac{n_2}{2}\tau^2 + \frac{n_3}{2}a^2. \qquad \textbf{(A.2)}$$

Differentiating W with respect to time t along the linearize system (A.1), we get

$$\frac{dW}{dt} = -w_{11}b^2 + w_{12}bp + w_{13}b\tau + w_{24}ap + w_{23}p\tau + w_{34}a\tau - w_{22}p^2 - w_{33}\tau^2 - w_{44}a^2. \qquad \textbf{(A.3)}$$

Where

$$w_{11} = \frac{rB^*}{K}\; w_{12} = \alpha\left(n_1 P^* - B^*\right),\; w_{13} = m\alpha\left(n_2 T^* - B^*\right),\; w_{22} = \frac{sn_1 P^*}{K_1},\; w_{33} = n_2\left(\mu_2 - m\alpha B^*\right),$$
$$w_{44} = n_3\eta_1,\; w_{24} = n_3\eta - \gamma n_1 P^*,\; w_{34} = n_2\gamma P^* - n_3\eta_0,\; w_{23} = \gamma n_2 A^*.$$

Now choosing $n_1 = \dfrac{B^*}{P^*}, n_2 = \dfrac{B^*}{T^*}, n_3 = \dfrac{\gamma P^* B^*}{\eta_0 T^*}$. **(A.4)**

Using (A.4), equation (A.3) reduce in following form

$$\frac{dW}{dt} = -w_{11}b^2 + w_{24}ap + w_{23}p\tau - w_{22}p^2 - w_{33}\tau^2 - w_{44}a^2. \qquad \textbf{(A.5)}$$

Sufficient conditions for $\dfrac{dW}{dt}$ to be negative definite are that the following inequalities hold:

$$\left(\gamma A^*\right)^2 < \frac{2sT^*}{K_1}\left(\mu_2 - m\alpha B^*\right), \qquad \textbf{(A.6)}$$

$$\left(P^* - \eta_0 T^*\right)^2 < \frac{2s\eta_1}{K_1\eta_0\gamma T^*}. \qquad \textbf{(A.7)}$$

This completes proof of Theorem (5.1).

Appendix B. Proof of Theorem (5.2)

For finding the condition of global stability as E_3 we construct the Lyapunov function

$$H = B - B^* - B^* \ln\frac{B}{B^*} + c_1\left(P - P^* - P^* \ln\frac{P}{P^*}\right) + \frac{c_2}{2}\left(T - T^*\right)^2 + \frac{c_3}{2}\left(A - A^*\right)^2. \quad \textbf{(B.1)}$$

Differentiating H with respect to time t along the solutions of model (1), we get

$$\frac{dH}{dt} = \frac{\left(B - B^*\right)}{B}\frac{dB}{dt} + \frac{c_1\left(P - P^*\right)}{P}\frac{dP}{dt} + c_2\left(T - T^*\right)\frac{dT}{dt} + c_3\left(A - A^*\right)\frac{dA}{dt}. \quad \textbf{(B.2)}$$

Using a system of equation (1), we get after some algebraic manipulations as

$$\begin{aligned}\frac{dH}{dt} = & -\frac{r}{K}\left(B - B^*\right)^2 - \frac{sc_1}{K_1}\left(P - P^*\right)^2 - c_2\left(\mu_2 - m\alpha B^*\right)\left(T - T^*\right)^2 - c_3\eta_1\left(A - A^*\right)^2 \\ & + \left\{- m\alpha + m\alpha c_2 T\right\}\left(B - B^*\right)\left(T - T^*\right) + \left(\gamma c_2\ P^* - c_3\eta_0\right)\left(T - T^*\right)\left(A - A^*\right) + \\ & \left(c_3\eta - c_1\gamma\right)\left(A - A^*\right)\left(P - P^*\right) + \left(c_1\alpha - \alpha\right)\left(B - B^*\right)\left(P - P^*\right) + \gamma c_2\ \mathrm{A}\left(T - T^*\right)\left(P - P^*\right).\end{aligned} \quad \textbf{(B.3)}$$

For simplicity, we choose $c_1 = 1, c_2 = \dfrac{\eta_0}{P^*\eta}, c_3 = \dfrac{\gamma}{\eta}$. **(B.4)**

Using (B.4), equation (B.3) can be written in following form

$$\begin{aligned}\frac{dH}{dt} = & -\frac{r}{K}\left(B - B^*\right)^2 - \frac{sc_1}{K_1}\left(P - P^*\right)^2 - c_2\left(\mu_2 - m\alpha B^*\right)\left(T - T^*\right)^2 - c_3\eta_1\left(A - A^*\right)^2 \\ & + \left\{- m\alpha + m\alpha c_2 T\right\}\left(B - B^*\right)\left(T - T^*\right) + \gamma c_2\ \mathrm{A}\left(T - T^*\right)\left(P - P^*\right).\end{aligned} \quad \textbf{(B.5)}$$

A sufficient condition for $\dfrac{dH}{dt}$ to be negative definite are that the following inequalities hold,

$$m^2\alpha^2\left(\eta_0 T_m - P^*\eta\right)^2 < \frac{2r\eta_0\eta P^*}{K}\left(\mu_2 - m\alpha B^*\right), \quad \textbf{(B.6)}$$

$$\gamma^2 A_m^2 < \frac{2s\eta P^*}{K_1\eta_0}\left(\mu_2 - m\alpha B^*\right). \quad \textbf{(B.7)}$$

Thus the model (2.1) is globally stable in the region Ω .

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